9. W. M. Yim and A. Amith, "Bismuth—antimony alloys for magnetothermoelectric cooling," Solid-State Electron., 15, 1141-1165 (1972).
10. H. J. Goldsmid, "Thermomagnetic energy conversion," Electron. Rev., 176, 514-518 (1965).
11. V. G. Okhrem and A. G. Samoilovich, "Physical principles of operation of a galvanothermomagnetic cooling element in the shape of a circular cylinder," Ukr. Fiz. Zh., 22, 39-44 (1977).
12. V. S. Vinnik and V. G. Okhrem, "Method of drift velocities and effect of inversion of the magnetic field on the thermal emf of bismuth," Fiz. Tverd. Tela, 18, 3411-3415 (1976).
13. G. M. Fikhtengolts, Study Course in Differential and Integral Calculus [in Russian], Vol. 3, Fizmatgiz, Moscow (1963).
14. Ya. I. Frenkel', Electrodynamics [in Russian], Vol. 2, ONTI, Moscow—Leningrad (1935).
15. A. G. Samoilovich and L. L. Korenblit, "Present status of the theory of thermoelectric and thermomagnetic effects in semiconductors," Usp. Fiz. Nauk, 49, 243-272 (1953).

OPTIMIZATION OF CURRENT LEAD WITH STRONG THERMAL INTERACTION WITH SURROUNDING STRUCTURAL ELEMENTS

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UDC 536.483

The article theoretically investigates the problem of optimizing a current lead according to the minimum heat flux at the cold end upon strong thermal interaction with the surrounding structural elements. It presents a number of generalized dependences characteristic of the optimum system.

In creating superconducting cryomagnetic systems for attaining high economic indicators, it becomes necessary to substantially reduce energy expenditure for compensating heat influxes into the cold zone. A number of authors [1-3] investigated in sufficient detail the problem of optimizing current leads without taking into account the effect of the structural elements surrounding them. However, it was shown in [4] that the temperature profiles along the current lead and the surrounding courses are fairly close to each other, which indicates considerable mutual thermal influence. The present article investigates the problems of optimizing current leads when there is considerable thermal interaction. As the initial assumption we use the assumption of ideal heat exchange, i.e., equal temperature profiles of the current lead, the cooling gas, and of the courses surrounding it which, according to [4], corresponds to a broad range of heat-transfer coefficients.

Let us examine the steady-state univariate equation of heat balance in the dimensionless form

$$\frac{d^2\Theta}{dx^2} = C\Theta_0' \frac{d\Theta}{dx} - \frac{1}{A} \cdot \frac{(\beta/\alpha)^2}{\beta/\alpha + 1}, \tag{1}$$

where

$$A = \frac{\Delta T}{\rho \dfrac{L}{S_{\text{т}}} \alpha} \cdot \frac{\beta}{\alpha}$$

is the dimensionless complex characterizing the thermal influence of the structural elements and is determined by the ratio of the amount of thermal energy transmitted through the structural elements by heat conduction to the amount of Joule heat released on the current lead whose thermal resistance is equal to the thermal resistance of the structural elements;

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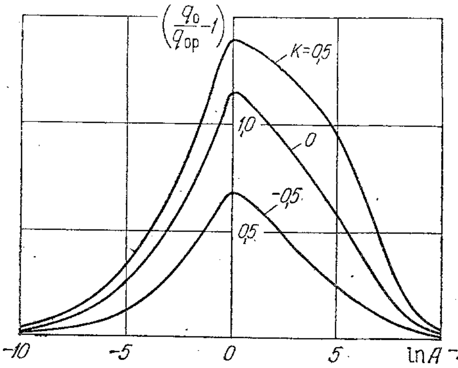

Fig. 1

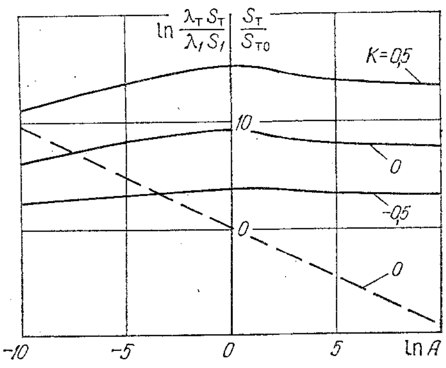

Fig. 2

Fig. 1. Dependence of the ratio of the heat fluxes ($q_{0T}/q_{0p}-1$) on ln A for different K.

Fig. 2. Dependence of the ratio of thermal resistances $\ln\frac{\lambda_T S_T}{\sum_{i=1}^{n}\lambda_i S_i}$ for different K (solid curves) and of the ratio of the cross-sectional areas S_T/S_{T0} for K = 0 (dashed curves) on ln A.

$$C=\frac{c_p\Delta T}{r}\left(1+\frac{\Phi}{q_0}\right)$$

is the dimensionless complex characterizing the thermophysical properties of the cooling vapors and the relative power of the additional heat influxes.

Boundary conditions:

$$x=0,\ \Theta=0;\ x=1,\ \Theta=1. \tag{2}$$

Flow rate of cooling vapor

$$m=\frac{C}{c_p}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\Theta_0'. \tag{3}$$

Heat flux at the cold end

$$q_0=\Delta T\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\Theta_0'. \tag{4}$$

If we solve Eq. (1), taking (2) into account, we obtain an expression for determining the dimensionless temperature gradient at the cold end of the system:

$$\Theta_0'=\frac{C\Theta_0'-\frac{1}{A}\frac{(\beta/\alpha)^2}{\beta/\alpha+1}}{\exp C\Theta_0'-1}+\frac{1}{A}\cdot\frac{(\beta/\alpha)^2}{(\beta/\alpha+1)\Theta_0'}. \tag{5}$$

We determine the geometric dimensions of the optimum current lead and the heat influxes at the cold end of the system from the joint solution of (4) and (5) and

$$\left[\frac{(\exp C\Theta_0'-1)\beta/\alpha}{C\Theta_0'(1+\beta/\alpha)}-\frac{C\Theta_0'[C\Theta_0'-(\beta/\alpha)^2/A(\beta/\alpha+1)]}{(\beta/\alpha+1)(1+\exp(-C\Theta_0'))}-1\right]\frac{1}{\exp C\Theta_0'-1}=0, \tag{6}$$

obtained for $\left.\frac{dq_0}{d\beta}\right|_{\alpha=\mathrm{const}}=0$. In case A = 0 ($\alpha=0$), it follows from (4)-(6) that

$$q=\sqrt{q\lambda_T\Delta T/C} \tag{7}$$

and

$$\frac{L}{S_T}=(C+1)\sqrt{\lambda_T\Delta T/\rho C} \tag{8}$$

for $C\gg 1$, which coincides with the result of [1] for a single current lead ($\alpha=0$).

For an analysis of the thermal processes in systems determined by (1) and (2), we plot the dependence of the ratio of the heat fluxes (Fig. 1) in optimizing, taking into account the effect of the structural elements (4)-(6) or disregarding them (7). It follows from (1) and (4)-(8) that $q_0/q_{0T} = F(C, A)$, i.e., that its form does not depend on the thermophysical properties of the materials and of the cooling vapors, and that the complexes A and C are the decisive parameters of the investigated system. The presented dependence ($C = 74$, $\Phi = 0$) shows that only when $A < 9 \cdot 10^{-4}$ or $A > 5 \cdot 10^4$, the mutual influence of the current lead and the structural elements may be neglected, and in the case $A < 1$ the current lead provides the main contribution to the heat balance whereas with $A > 1$ the decisive factor are the structural elements. The largest value of the ratio of the heat fluxes is attained in the region of $A = 1$, i.e., when $\frac{1}{\alpha} = \sqrt{\rho \frac{L/\Delta T}{S_T} \cdot \frac{1}{\beta}}$.

A characteristic feature of the system determined by (4)-(6) is the existence of a positive heat influx from the hot end, and this increases, beginning from the zero value with $A = 0$, and attains its maximum value determined by the thermal resistance of the structural elements ($A > 5 \cdot 10^4$).

An analysis of the dependences of the ratios of the heat fluxes at the ends of the system shows that in the region $A < 1$, where the current lead is decisive, the optimum section of the current lead increases when the influence of the structural elements increases (Fig. 2), and consequently, the amount of heat liberated in the system due to Joule losses decreases. The process in question occurs more intensively than the relative increase of the heat flux from the hot end into the system, and this leads to decreased heat influx at the cold end. In the region $A > 1$ the heat influx is basically determined by the total thermal resistance of the system because the contribution of energy by Joule heat released to the balance of the system becomes small. Therefore the optimum cross section of the current lead with increasing A decreases and tends toward a constant value when $A > 5 \cdot 10^4$, i.e., in this case the current lead has practically no effect on the system. As can be seen from Fig. 2, when $A = 1$, the heat fluxes along the structural elements and the current lead are equal, and hence follows that the flux of Joule heat has the same magnitude. The region $A \sim 1$ is the region of the greatest thermal influence.

If we examine the above dependences (Figs. 1, 2) with different values of the complex C, we note that an increase in mass expenditure (by additional heat influxes) leads to an increased effect of mutual influence of the current lead and the structural elements while the characteristic features of the process of heat exchange in the system are maintained, and this finds expression in the increase of the maximum values of the relative heat influxes and the increased range of the values A at which the mutual influence is substantial. In view of the dependence of the complexes A and C on the temperature difference at the ends of the system, we may assert that it has the strongest influence on the interaction effect, and that its increase is equivalent to the simultaneous increase of the relative additional heat influxes (Φ/q_0) and the decrease of the thermal resistance of the structural elements. This conclusion is confirmed by the results of experimental work with real structures of cryogenic systems which show the following. When particularly thin-walled courses are used for forming the channel of the current lead, and there are large current loads ($I > 1$ kA), the heat influx at the cold end depends only weakly on the temperature of the hot end; this is a characteristic of a single current lead, but in the case of real structures of cryostats for superconducting magnetic systems, where $I < 1$ kA, the effect of the lowered temperature of the hot end of the system current lead—course up to the level of 80°K is substantial.

To determine the energy characteristics, we will analyze the irreversible losses occurring when the system (1)-(2) is realized. The irreversible losses were determined as the difference between the minimum energy expenditures [5] for maintaining the temperature regime of the investigated system and the sum of the minimum energy expenditures in the case of realization of two systems within a single current lead and structural elements that are not interconnected by heat-exchange processes. It was assumed that $\Phi/q_0 = \text{const}$ for the systems under comparison. It follows from (1) and (4)-(8) that $(E - E_p)/E_p = f(C, A)$, i.e., that it is also a characteristic dependence (Fig. 3) for the investigated system.

The presented curves (Fig. 3) enable us to state that the irreversible losses are determined by the magnitude of the mutual influence of the elements contained in the system,

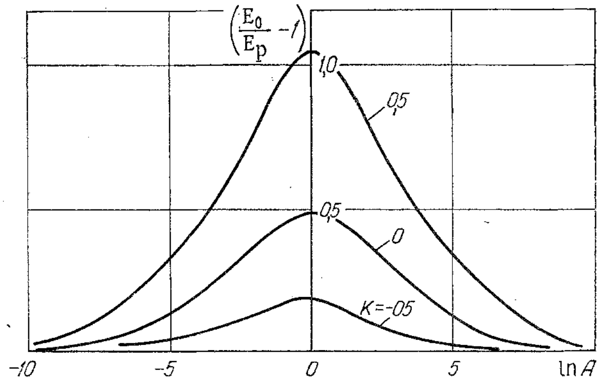

Fig. 3. Dependence of the ratio of energy expenditures $(E_0/E_p - 1)\cdot 10$ on ln A for different K.

and that their maximum corresponds to the equality of the heat fluxes in the current lead and the structural elements.

An analysis of the dependences (Figs. 1, 2) shows that the source of the irreversible losses is the process of heat exchange between the elements of the system. In view of the dependence of the minimum energy expenditure on the heat influx at the cold end [5], we may conclude from Figs. 1, 2 that in the case of maximum mutual influence (A = 1), the unification of the structural elements and the single current lead into a single thermal system makes it indispensable to increase the minimum energy expenditures by 125%. In the realization of a system including the current lead, whose optimization was carried out by taking into account the effect of the structural elements (4), (6), the irreversible losses with A = 1 amount to 5%. In view of the fact that in the process of heat exchange, the structural elements are heated by part of the Joule heat liberated on the current lead, it follows from a comparison of the quoted values of minimum energy expenditures and Joule losses that the cause of the substantial reduction in energy expenditures in optimization (4), (6) is the intense decrease of thermal interaction in the system.

The calculation of the geometric dimensions of the current lead, taking the influence of the structural elements in the case of weak dependence of the physical properties of the material on the temperature into account, consists of the following stages.

Proceeding from the magnitude of the thermal resistance of the courses, and also of the other elements that interact thermally with the current lead, of the additional heat influx and the current intensity determined by the design of the cryomagnetic device, we find the values of the complexes A and C.

From the expressions (7) and (8) we find the geometric dimensions and heat influxes for a single current lead.

The heat flux at the cold end of a system that differs from the specified one by the lack of a current lead is obtained from the solution of Eq. (1):

$$q_{0k} = \frac{\ln(C+1)}{C}\Delta T_\alpha$$

and from summing with the heat flux along a single current lead.

The geometric dimensions of the optimum current lead and the heat flux at the cold end of the specified system are determined by proceeding from the dimensions of a single optimum current lead and the summary heat influx, multiplying them respectively by the coefficient of increased cross section (Fig. 2) and the irreversible losses $(E - E_p)/E_p = f(A, C)$ (Fig. 3).

When the physical properties of the materials are strongly dependent on the temperature, the calculation is carried out in the following sequence.

We specify the mean integral temperature, and from it we calculate the values of the complexes A and C.

Proceeding from the above dependences, we determine the coefficients of increased cross section of the current lead (Fig. 2) and of the irreversible losses (Fig. 3).

By solving the equation of the heat balance for a system differing from the specified one by the absence of a current lead, and by the joint solution of analogous equations for a single current lead with the condition of its optimality [1-3], we find the cross section of the optimum single current lead and the summary heat flux at the cold end.

The further calculation consists in solving the equations of heat balance of the overall system jointly with the condition of optimality $dq/dS_T = 0$, where we use as initial approximation the previously determined values of the cross section of the current lead and of the heat influx, taking the respective coefficients of increase into account.

Finding the initial approximation by the method shown above is obligatory because the equation $dq/dS_T = 0$ may have up to three roots, and one of them realizes the system with zero heat influx at the hot end. In this case the geometric dimensions of the current lead and the heat influx are determined from (7), (8), with the coefficient of heat conduction of the material being replaced by the reduced coefficient of heat conduction of the system $\lambda_{re} = \lambda_T(1 + \beta/\alpha)$. It is obvious that with $A \to \infty$ the cross section of the current lead tends toward zero, and the heat influx at the cold end increases intensely as a result of the unbounded increase of Joule losses.

A system in which the heat influxes at the ends are equal also realizes the condition $dq_0/dS_T = 0$, however, here too the heat flux at the cold end is not the smallest possible one.

As an example we will present the data of the calculation using the presented optimization method. The physical properties were assumed to be dependent on the temperature. As structural elements we took a course of 0.1-m diameter and wall thickness of $6 \cdot 10^{-4}$ m, made of steel Kh18N10T. The material of the current lead was brass L62. Putting the mean integral temperature equal to 20°K, we obtain $A \cdot I^2 = 192\ A^2$, $C = 73.4$ ($\Delta T = 295.2$°K, $\Phi = 0$), i.e., in the interval $I < 460$ A and $I > 6 \cdot 10^{-2}$ A, the interaction effect is considerable. With a feed current of the superconducting solenoid equal to 100 A, we have from Fig. 3 that $(E - E_p)/E_p = 2 \cdot 10^{-2}$, i.e., the sum of the minimum energy expenditures and correspondingly the sum of heat influxes at the cold end are practically equal in both cases. As a result of solving the equations of the heat balance for the separate systems, $q_p = 1.54 \cdot 10^{-3}$ W/A. If we use the found value of the heat flux as initial approximation and solve the equation of the heat balance for the sought system with minimization of the heat flux at the cold end by changing the cross section of the current lead, we finally obtain the optimum cross section of the current lead $S_T = 8.8 \cdot 10^{-7}\ m^2/A$ and the heat influx $q_0 = 1.59 \cdot 10^{-3}$ W/A.

The slight difference between the values taken as initial approximation and those obtained as a result of the final calculation is chiefly due to the correct selection of the mean integral temperature, and it leads to a substantial reduction of the time required for computer calculation.

NOTATION

x, dimensionless coordinate; L, length of the current lead; T, current temperature; $\Delta T = T(L) - T(0)$, temperature difference at the ends of the system; $\Theta = (T - T(0))/\Delta T$, dimensionless temperature; $\Theta_0' = d\Theta/dx|_{x=0}$, dimensionless temperature gradient at the cold end; q_0, heat flux at the cold end of the system with a current of 1 A; m, mass flow rate of the cooling vapor with a current of 1 A; Φ, additional heat influx to liquid helium; $K = \Phi/q_0$, reduced additional heat influx to liquid helium; S_T, cross-sectional area of the current lead with a current of 1 A; S_{T_0}, optimum cross-sectional area of single current lead with a current of 1 A; q_{cT}, heat flux at the cold end of the system with a current equal to 1 A, and a current lead whose geometric dimensions are determined without taking the effect of the structural elements into account; λ_T, thermal conductivity of the material of the current lead; S_i, cross-sectional area of the structural element per unit current flowing through the current lead; λ_i, thermal conductivity of the material of the structural element; $\alpha = L/\sum_{i=1}^{n} \lambda_i S_i$, summary thermal resistance of the structural elements; $\beta = L/\lambda_T S_T$, thermal resistance of the current lead; I, current flowing through the current lead; r, evaporation heat of helium; ρ, electric resistivity of the material of the current lead; c_p, specific heat of the cooling vapors; E, minimum energy expenditures for maintaining the temperature regime of the system with a current of 1 A; E_p, sum of the minimum energy expenditures in the realization of two separate systems; q_{op}, minimum summary heat flux at the cold end in the realization of two separate systems.

LITERATURE CITED

1. J. M. Lock, "Optimization of current leads into a cryostat," Cryogenics, December, 438-442 (1969).
2. A. G. Shifel'bain, S. P. Gorbachev, and V. K. Matyushchenkov, "Special features of sectional current leads," in: Superconductivity, Vol. 5, Atomizdat, Moscow (1977), pp. 65-68.
3. V. K. Litvinov, V. I. Kurochkin, and V. I. Karlashchuk, "Sectional optimized current leads," Inzh.-Fiz. Zh., 37, No. 2, 360-365 (1979).
4. A. S. Veselovskii, V. I. Gemintern, E. V. Lebedeva, S. S. Skvortsov, and E. S. Sklyar, "Calculation of the heat influxes in cooling two structural elements of cryostats with a common gas stream," in: Proceedings of the All-Union Electromechanical Research Institute, Moscow (1977), pp. 68-75.
5. M. P. Malkov (editor), Handbook of the Physicotechnical Foundations of Cryogenics [in Russian], Énergiya, Moscow (1973).

DETERMINATION OF THE OPTIMUM DIMENSIONS OF COOLING CHANNELS

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UDC 62-713

The article presents the formulas for calculating the optimum equivalent diameter of cooling channels in devices with high specific thermal loads.

In designing cooling systems of the most variegated machines and devices with high specific thermal loads, the most important problems are reduction of the weight and size and lowering of the energy requirements of liquid-cooling systems (LCS). An effective way of solving these problems is the optimization of the dimensions of the cooling channels.

As a rule, the liquid circuit of an LCS consists of the cooling channels of the device (machine), of communication pipes, and of channels of the heat exchanger.

The power required for pumping the heat carrier through the heat exchanger and the communication pipes is usually much lower than the power required for pumping the heat carrier through the cooling channels of the devices, and therefore the energy consumption of the LCS can be minimized chiefly by optimizing the dimensions of the cooling channels.

In the present work, the power consumption was optimized with constant mean temperature of the liquid in the cooling channels of the device and of the heat exchanger. Minimization of the power, required for pumping the heat carrier through, leads to reduced weight and size of the pump, with unchanged weight and size of the other elements of the cooling system.

Such a statement of the problem of optimizing the dimensions of the cooling channels ensures the minimization of the fundamental optimization criteria accepted at present in engineering practice the world over [1, 2]: the weight of the complex, the volume of the complex, the costs of producing it, and operating costs.

The object of the present work is to obtain dependences making it possible to determine the optimum equivalent diameter of the cooling channels and the required pump head corresponding to it ensuring with the specified values of the flow density on the surface of the cooling channel, the total surface of the cooling channels, the length of the cooling

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